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Problem Set 1

Section 1.1

1) - We proved in class that $|z_1 z_2| = |z_1||z_2|$
   - We also know that the only complex number with magnitude equal to 0 is zero

$\therefore$ If $z_1 z_2 = 0$, then $|z_1 z_2| = 0$

$\boxed{\text{PROOF}}$ $|z_1 z_2| = 0$

$|z_1||z_2| = 0$

↳ The only solutions to this equation are

(1) $|z_1| = 0$

(2) $|z_2| = 0$

(3) $|z_1|$ and $|z_2| = 0$

- If $|z_1| = 0$, $z_1 = 0$. If $|z_2| = 0$, $z_2 = 0$

$\therefore$ If $z_1 z_2 = 0$, $z_1 = 0$ and/or $z_2 = 0$

2) $\dfrac{(8+2i)-(1-i)}{(2+i)^2} = \dfrac{8+2i-1+i}{(2+i)(2+i)} = \dfrac{7+3i}{(2+i)(2+i)}$

$(2+i)(2+i)$
$= 4+4i-1$
$= 3+4i$

$\dfrac{(7+3i)(3-4i)}{(3+4i)(3-4i)} = \dfrac{21-28i+9i+12}{9-12i+12i+16} = \dfrac{33-19i}{25}$

$= \boxed{\dfrac{33}{25} - \dfrac{19}{25}i}$

3) $\frac{2+i}{6i-1+2i} = \frac{2+i}{-1+8i} \rightarrow \frac{(2+i)(2+i)}{(-1+8i)(-1+8i)} = \frac{4+4i-1}{1-16i-64} = \frac{3+4i}{-63-16i}$

$$\frac{(3+4i)(-63+16i)}{(-63-16i)(-63+16i)} = \frac{-189+48i-252i-64}{3969+256} = \frac{-253-\overset{204}{\cancel{300}}i}{4225} = \boxed{\frac{-253}{4225} - \frac{204}{4225}i}$$

$$= \cancel{\frac{-253}{4225}} \cancel{-\frac{204}{205}}$$

4) $(2+i)(-1-i)(3-2i)$

$= (-2-2i-i+1)(3-2i)$

$= -6+4i-6i-4-3i-2+3-2i$

$= \boxed{-9-7i}$

Section 1.2

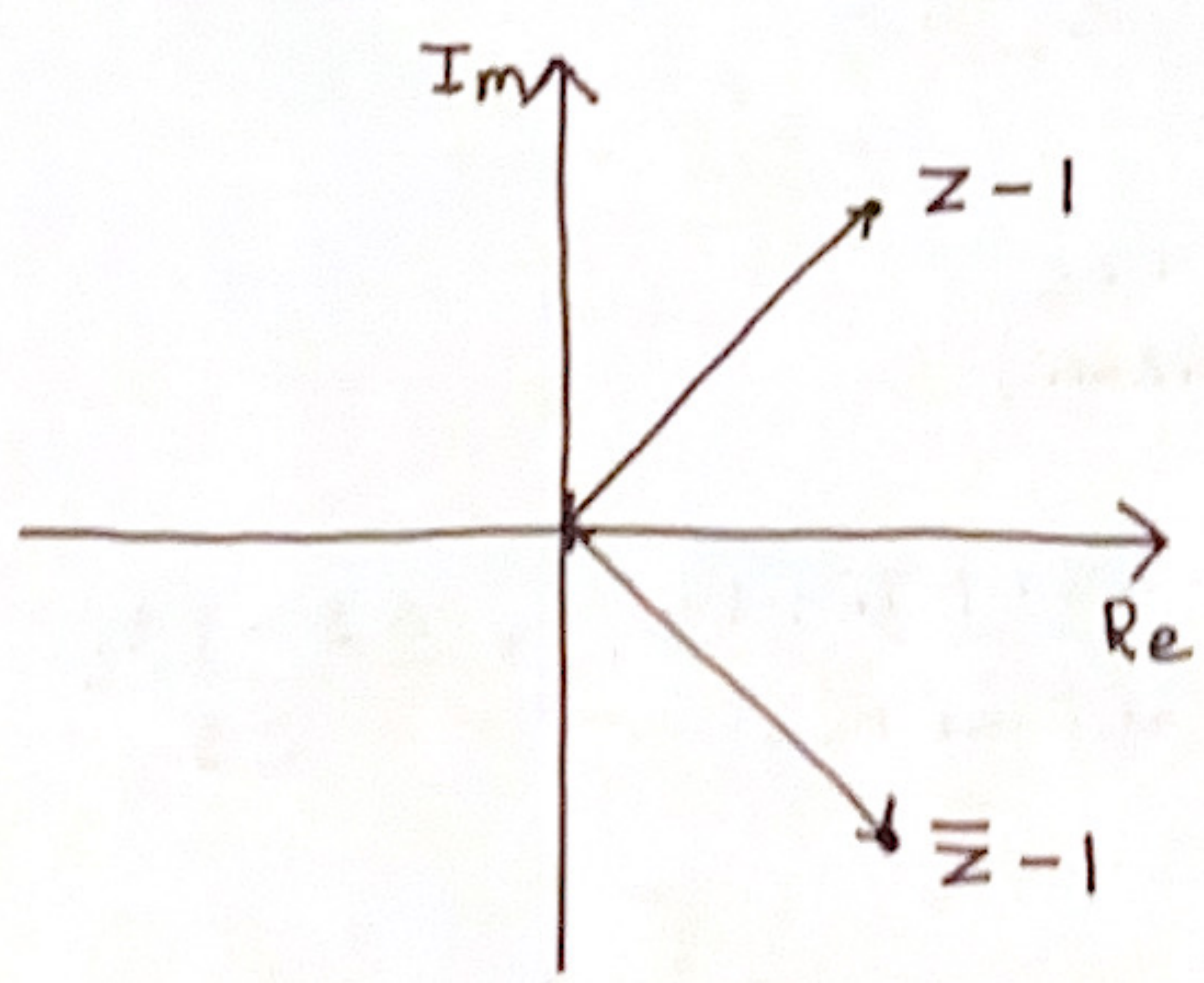
1) $|z-1| = |\bar{z}-1|$

$|a+bi-1| = |a-bi-1|$

$\sqrt{(a-1)^2+(bi)^2} = \sqrt{(a-1)^2+(-bi)^2}$

$(a-1)^2+(bi)^2 = (a-1)^2+(-bi)^2$

$\boxed{(a-1)^2-b^2 = (a-1)^2-b^2}$



- $\bar{z}$ is just z reflected in the real axis. Because subtracting 1 just moves z and $\bar{z}$ left on the Real axis, it can be seen clearly that they're magnitudes have to be equal.

2) $(\bar{z})^2 = (z)^2$

$(a-bi)^2 = (a+bi)^2$

$(a-bi)(a-bi) = (a+bi)(a+bi)$

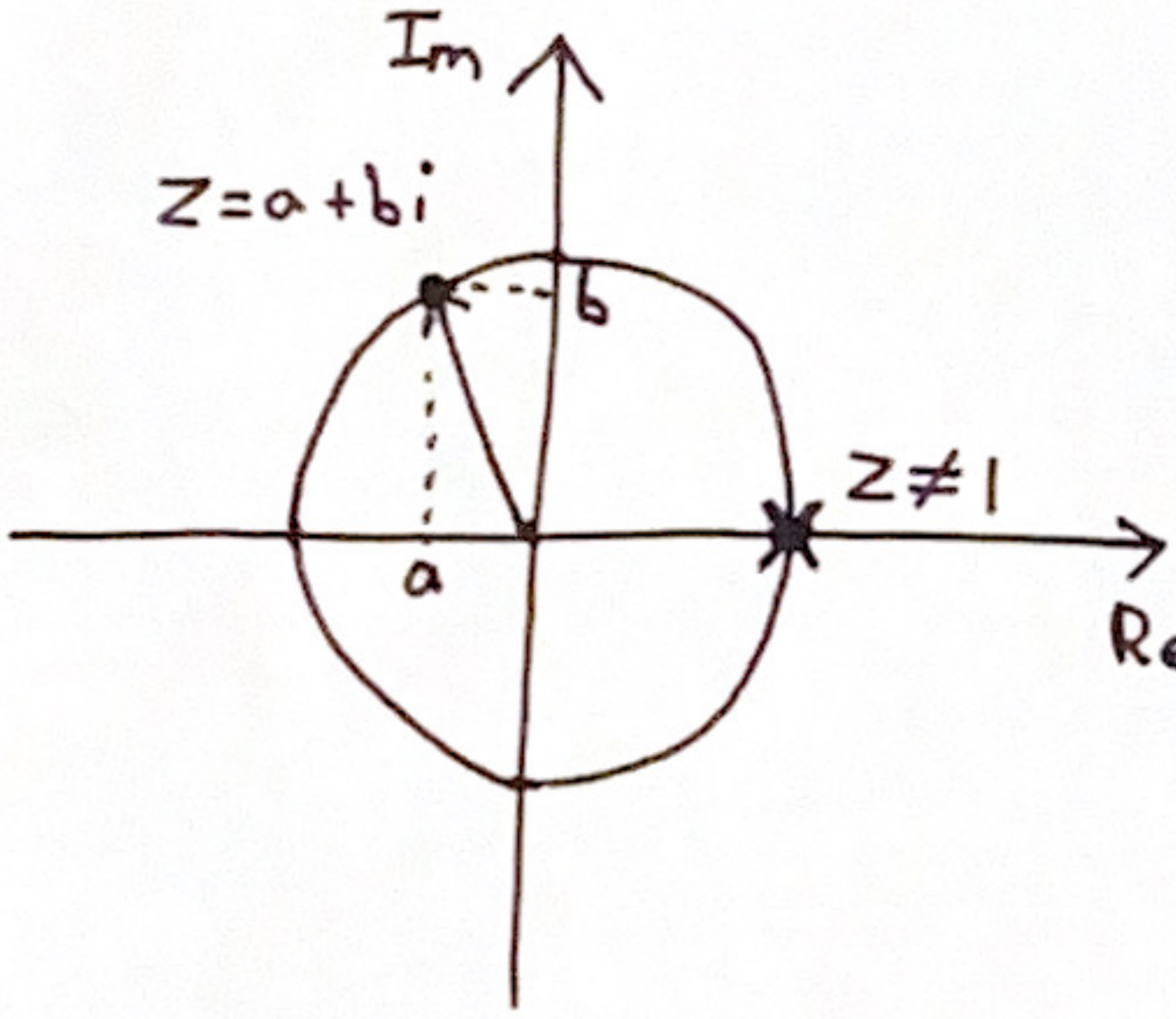
$a^2 - 2abi - b^2 = a^2 + 2abi - b^2$

$(a^2-b^2) - 2abi = (a^2-b^2) + 2abi$

For these equations to be equal, the 2abi term must go to zero. This means that <u>either</u> a or b must be 0.

$\therefore$ Z is either pure Real or Pure Imaginary

3) $|z| = 1$ means that we are on the Unit Circle



$|z| = 1$

$\sqrt{a^2+b^2} = 1$

$a^2+b^2 = 1$

$a^2 = -b^2$

$\frac{1}{1-z} = \frac{1}{1-a-bi} = \frac{1}{(1-a)-bi}$

$=$ [illegible]

$= \frac{(1-a)+bi}{1-2a+a^2+b^2}$

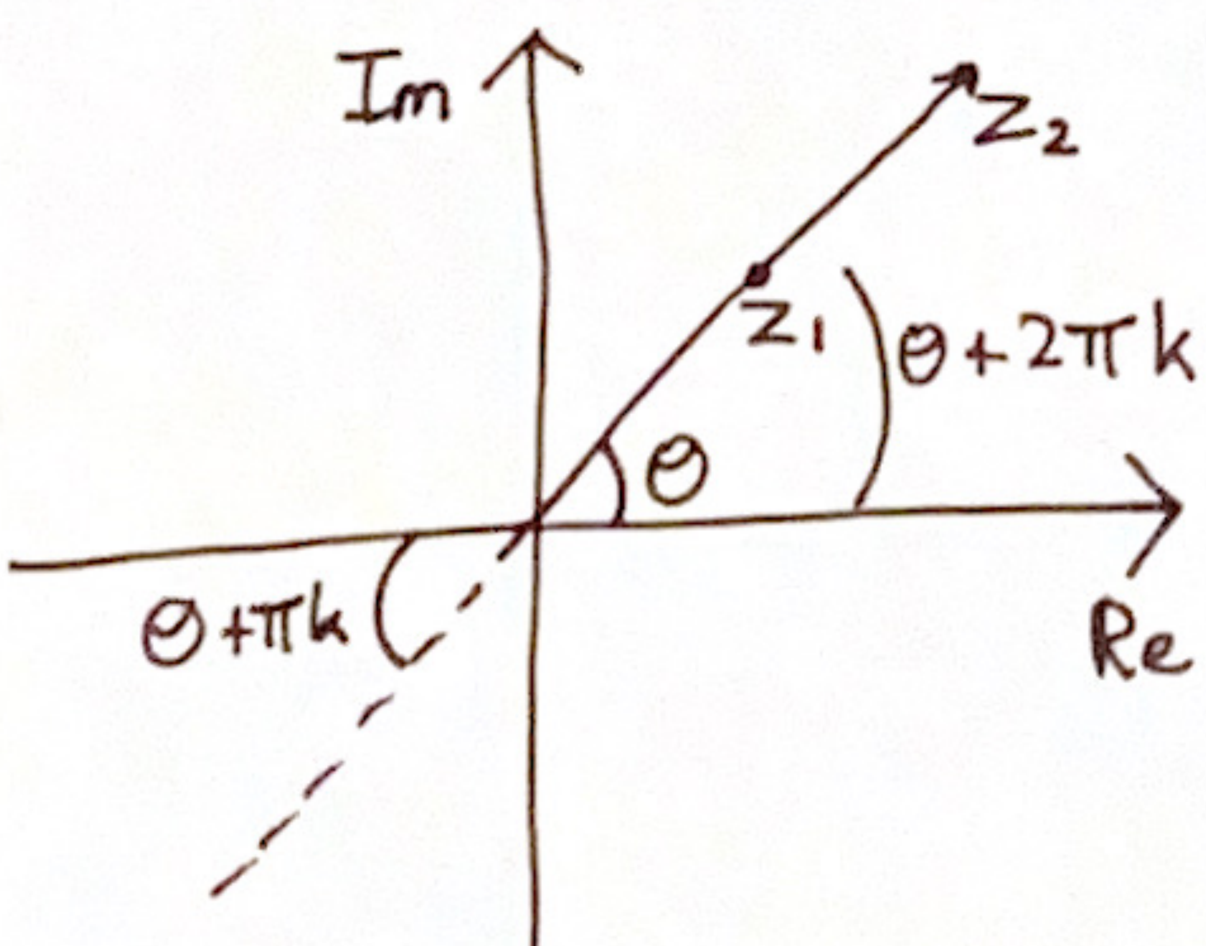
$= \frac{(1-a)+bi}{1-2a+a^2-a^2}$

$= \frac{1-a}{1-2a} = \frac{1}{2}$

$Re\left[\frac{(1-a)+bi}{1-2a+a^2-a^2}\right]$

$\boxed{\text{Section 1.3}}$

1) z_1 is parallel to z_2 if their angles differ by a multiple of $k\pi$, where k is an Integer.



$(a_1+b_1i)(a_2-b_2i)$

$a_1a_2 - a_1b_2i + a_2b_1i + b_1b_2$

$Im(a_1a_2 - a_1b_2i + a_2b_1i + b_1b_2)$

$= i(-a_1b_2 + a_2b_1) = 0$

$a_1b_2 = a_2b_1$

2) $\bar{z}_1 z_2 = (x_1 - y_1 i)(x_2 + y_2 i)$

$= x_1x_2 + x_1y_2 i - x_2y_1 i + y_1y_2$

$Re(x_1x_2 + x_1y_2 i - x_2y_1 i + y_1y_2) = \boxed{x_1x_2 + y_1y_2 = V_1 \cdot V_2}$

Section 1.4

1) (a) $1 \cdot \left(e^{i2\pi/9}\right)^3 = \boxed{e^{i2\pi/3}}$

(b) $\frac{2+2i}{-\sqrt{3}+i} \to \frac{(2+2i)(-\sqrt{3}-i)}{(-\sqrt{3}+i)(-\sqrt{3}-i)} = \frac{-2\sqrt{3}-2i-2\sqrt{3}i+2}{3-1}$

$= -\sqrt{3} - i - \sqrt{3}i + 1$

$= 1-\sqrt{3} + (-1-\sqrt{3})i$

$\theta = \frac{-7}{12}\pi$

$r = \sqrt{2}$

$\boxed{\sqrt{2}e^{i\left(\frac{-7\pi}{12}\right)}}$

(c) $\frac{2i}{3e^{(4+i)}} \to 2i = 2e^{i\pi/2}$

$= \frac{2e^{i\pi/2}}{3e^4 \cdot e^i} = \boxed{\frac{2}{3e^4} \cdot e^{i(\pi/2 - 1)}}$

2) (a) $e^{z+\pi i} = e^z \cdot e^{\pi i} = e^z \cdot (-1) = \boxed{-e^z}$

(b) $\overline{(e^z)} = \overline{(e^a \cdot e^{bi})} = e^a \cdot e^{-bi} = e^{a-bi} = \boxed{e^{\bar{z}}}$

3) $\boxed{\text{YES}}$

$$(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin\theta$$

$$(e^{i\theta})^n = \underbrace{e^{i\theta}\cdot e^{i\theta}\cdot\ldots\cdot e^{i\theta}}_{n\text{ times}}$$, for $n \in \mathbb{Z}$. No restriction on negative integers

$$e^{i\theta n} = e^{i\theta n}$$

4) ~~xxx~~ $z = re^{i\theta}$, ~~xxx~~

$e^{\ln(r)} = r$, therefore we can substitute that in for r

$\downarrow$

$$z = e^{\ln(r)}\cdot e^{i\theta} = \boxed{e^{\ln(r)+i\theta}}$$

$\boxed{\text{Section 1.5}}$

1) $(\sqrt{3}-i)^7$, $n=7$
$r=2$
$\theta = -\pi/6$

$$2^7\left[\cos\left(\tfrac{-7\pi}{6}\right) + i\sin\left(\tfrac{-7\pi}{6}\right)\right] = \boxed{-64\sqrt{3}+64i}$$

(b) $(1+i)^{95}$, $n=95$
$r=\sqrt{2}$
$\theta = \pi/4$

$$\sqrt{2}^{95}\left[\cos\left(\tfrac{95\pi}{4}\right) + i\sin\left(\tfrac{95\pi}{4}\right)\right] = \boxed{2^{47}(1-i)}$$

2) $z^4 + 1 = 0$

$z_k = \sqrt[n]{r}\, e^{i\frac{\theta + 2\pi k}{n}}, \quad 0 < k < n-1$

$z_0 = \sqrt[4]{1}\, e^{i(\pi/4)} = \boxed{\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} i}$

$z_1 = \sqrt[4]{1}\, e^{i(\frac{\pi}{4} + \frac{2\pi}{4})} = \boxed{-1/\sqrt{2} + 1/\sqrt{2}\, i}$

$z_2 = \sqrt[4]{1}\, e^{i(\pi/4 + \frac{4\pi}{4})} = \boxed{-1/\sqrt{2} - 1/\sqrt{2}\, i}$

$z_3 = \sqrt[4]{1}\, e^{i(\pi/4 + \frac{6\pi}{4})} = \boxed{1/\sqrt{2} - 1/\sqrt{2}\, i}$

Roots Match

$z^4 + 1 = (z^2 - \sqrt{2} z + 1)(z^2 + \sqrt{2} z + 1)$

$= \left(z + \frac{(-1+i)}{\sqrt{2}}\right)\left(z + \frac{(-1-i)}{\sqrt{2}}\right)\left(z + \frac{(1-i)}{\sqrt{2}}\right)\left(z + \frac{(1+i)}{\sqrt{2}}\right)$

3) $z = a + bi$

$(z+1)^{100} = (z-1)^{100}$

$(\cancel{a} + bi + 1)^{100} = (\cancel{a} + bi - 1)^{100}$

$(bi + 1)^{100} = (bi - 1)^{100}$

$\therefore \operatorname{Re}(z) = 0$